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Cite as: J. Appl. Phys. 125, 053905 (2019); https://doi.org/10.1063/1.5080449
Submitted: 08 November 2018. Accepted: 14 January 2019. Published Online: 05 February 2019

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Submitted: 8 November 2018 · Accepted: 14 January 2019 · Published Online: 5 February 2019

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ABSTRACT

We report experiments which characterize spin wave propagation in a thin (III) yttrium iron garnet film for arbitrary angles between the in-plane magnetic field and the mode wavevectors. By measuring the magnetic field evolution of the phase of the wave traveling across the film, we deduce the frequency dependence of the wavevector, the dispersion relation, from which the mode velocity follows. Additionally, we observe multiple nodes in the regime of the propagating Damon-Eshbach mode; these arise from avoided crossings associated with the higher, exchange split, standing wave modes along the film normal, the positions of which correlate with the direct absorption measurements of their positions. This information allows a determination of the exchange parameter. Using this technique, we examine the nonreciprocity in spin wave propagation that results from an adjacent metal layer.

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I. INTRODUCTION

The reading and writing of static domain structures in ferromagnets remains the basis for most high density information storage. Strategies to enhance this density continue to evolve; however, the ultimate limits are set by thermal fluctuations over various energy barriers defining the local spin orientation of an information bit. On the other hand, the speed with which data are exchanged with magnetic storage media is also limited by physical processes, e.g., the Larmor period, 1/ωL, characterizing the precession rate of elemental spins in achievable local magnetic fields.

In addition to various domain structures and their manipulation, propagating magnetic excitations, spin waves, may also have applications. Of particular interest from the device point of view has been the material yttrium iron garnet (YIG) with the chemical formula Y3Fe2(FeO4)3. YIG is a ferrite known for its narrow ferromagnetic resonance (FMR) line widths which have been exploited to make microwave filters. The associated low spin wave damping also suggests possible uses in delay lines (similar to surface acoustic wave devices) that operate in the GHz range and was an active topic in the 1970s.1–4 Micron scale single crystal YIG films can be grown on lattice matched gadolinium gallium garnet (GGG) substrates by liquid phase epitaxy that have line widths of a few tens of gauss, opening a path to thin film based devices.

For the last ten years or so, there has been a large activity centered on patterning various thin film periodic nanostructures, mostly from permalloy; the FMR spectra of such structures display, in addition to uniform precession, various standing spin wave modes dictated by sample geometries, e.g., the Brillouin zone centered “optical” modes generated by a periodicity. This and other phenomenology are now regarded as part of the emerging field of Magnonics.

Device patterning is not limited to periodic or isolated nanostructures. Looking ahead, one can imagine magnetic islands coupled by spin wave transmission lines designed to perform logic or other functions. Whether such devices can be competitive or find niche applications is not certain at this time. Were this to happen, a better characterization of spin wave propagation in materials such as YIG may prove essential. One problem is the extreme in-plane anisotropy spin wave propagation displays, which we will address below.
In any case, a better characterization of spin wave propagation is of interest from a fundamental point of view. While phase detection of spin waves has been investigated in the past, new developments in the theory of spin wave propagation have arisen that warrant additional work.

II. MAGNETOSTATIC MODES

In the magnetostatic limit, where the wave length of the spin excitations is short compared to that of an electromagnetic wave of the same frequency and the dipole interaction is larger than the exchange interaction, there exist three classes of modes which propagate in the plane of a ferromagnetic platelet and which correspond to different directions of the applied magnetic field. Two of these modes are distributed throughout the bulk of the material and are designated as volume modes, while the third is restricted to a surface, which we refer to as the Damon–Eshbach (DE) mode, after the investigators who first clarified its nature.

One of the bulk modes, the forward volume (FV) mode, propagates in the same direction (and hence its name) as its wavevector, \( \mathbf{k} \), and can be excited when the applied magnetic field, \( \mathbf{H} \), is largely perpendicular to the plane of the sample. The behaviors of this mode will be the focus of a separate work.

The DE mode can propagate when the applied magnetic field lies in the plane of the sample but largely perpendicular to the in-plane wavevector. This mode is a surface wave which has an imaginary component of the wavevector in the thickness direction, the magnitude of which increases with the in-plane component, \( k \). For \( ks \ll 1 \), with \( s \) the thickness of the film, the amplitude is effectively uniform in the thickness direction.

The second mode, the so-called backward volume (BV) mode, is a bulk or volume mode; it can be excited when \( \mathbf{H} \) is largely parallel to \( \mathbf{k} \). It derives its name from the unusual fact that the frequency decreases with increasing wavevector; i.e., the group velocity is negative.

Higher order BV modes are also allowed which involve standing waves in the thickness direction with discrete wavevectors. Due to the exchange interaction, which dominates the dispersion for small thicknesses (and hence large values of the discrete wavevectors), these modes are pushed to higher frequencies.

The dispersion relation for the geometries associated with the DE and BV modes in YIG for an applied field of 1402 G is shown in Fig. 1.

For both the DE and BV modes, the \( k = 0 \) spin wave corresponds to the ferromagnetic resonance (FMR) frequency, which for an in-plane applied magnetic field \( H_0 \) occurs at a frequency given by the Kittel equation

\[
\omega = \gamma \sqrt{H_0(0 + 4\pi M)};
\]

here, \( \gamma \) is the gyromagnetic ratio, \( 2\pi \times 2.8 \text{ MHz/G} \) obtained from a separate measurement, \( f \) is the microwave frequency, and \( M \) is the saturation magnetization of the sample. When scanning the applied field, as done in this work, the position of the FMR frequency will shift accordingly. This allows us to change the spin wave wavevector by varying the field at a fixed frequency. In the case where \( k \) is parallel to \( \mathbf{H} \), a field increase will increase the magnitude of \( \mathbf{k} \) as we follow the BV mode dispersion relation of Fig. 1. In the case of \( k \) perpendicular to \( \mathbf{H} \), while still in plane, a field decrease will increase the magnitude of \( \mathbf{k} \) as we follow the Damon–Eshbach mode dispersion relation of Fig. 1.

As noted above, higher order BV modes are also allowed which involve standing waves in the thickness direction with discrete wavevectors. Due to the exchange interaction, which dominates the dispersion for small thicknesses (and hence large values of the discrete wavevectors), these modes are pushed to higher frequencies and can lie above the ferromagnetic resonance frequency.

III. EXPERIMENTAL TECHNIQUES

The YIG film used in these experiments had a width of 5 mm and a length of 10 mm. It was supplied by MTI Corporation and was grown using liquid phase epitaxy on a (III) GGG substrate. The thickness of the sample was measured through ellipsometry to be \( 2.843 \pm 0.002 \mu m \). Earlier works involving this sample used the manufacturer’s stated thickness of \( 3.05 \mu m \).

To generate and detect the spin waves, we use the following technique:14,15 We place the substrate supporting our thin YIG film between two 50 \( \mu \)m wires as shown in Fig. 2.

Through the bulk/vacuum Maxwell boundary conditions, an
In our experiments, we apply 25 dBm of 6 GHz microwaves to one of the wires, and the magnetic field is then swept over an interval that includes the FMR resonance together with the neighboring oscillations arising from spin wave propagation. Depending on the magnitude and direction of the applied field and the microwave frequency, spin waves of different wavelengths are launched in accordance with Fig. 1.

The output of the receiving wire will be of the form $A(H) \exp(ikd)$, where $k(H)$ is the wavevector of the spin wave for a given applied magnetic field and $d$ is the width of the sample, 5 mm in our case; $A(H)$ depends on the wire/film coupling (constrained at short wavelengths by a lack of parallelness, edge roughness, and static field inhomogeneities associated with demagnetization effects). The phase of the output is then dependent on the applied magnetic field, thereby allowing us to detect the phase difference between the applied and the received signal using the microwave mixer. This change in phase then corresponds to a change in the wavevector of the spin waves. We also apply a 380 Hz field modulation to facilitate detection with a lock-in amplifier. The angle between the wavevector and field can also be varied by rotating the magnet around the sample normal keeping the field in the film plane.

**IV. Backward Volume Mode: Measurements and Comparison with Theory**

When the magnetic field is parallel to the in-plane wavevector, the backward volume mode propagates for a range of oscillating magnetic field generated by a microwave current of the appropriate frequency passing through the transmitting wire can launch a spin wave at the vacuum/ferrite interface which then propagates perpendicular to the interface (the microwave phase being taken as constant across the wire); the arriving spin wave generates a microwave current in the receiving wire at the opposing side of the film via the inverse effect, which is subsequently detected with a microwave mixer, thereby allowing us to measure the phase evolution of the arriving spin waves. The two opposing YIG/substrate interfaces are assumed flat and parallel; a lack of parallelness or flatness can result in a cancellation of the arriving wavefronts across the interface due to a varying path length and associated phase shift. A virtue of this approach is the absence of a geometric cutoff wavelength, unlike that associated with the finite strips placed on top of a sample, which is more commonly used to excite and receive spin waves.28

*Fig. 2.* A schematic of the experimental setup together with the sample showing the input and output coupling wires. The two 50 $\mu$m wires run adjacent to the 10 mm long edge of the sample with one end tied to the ground for a separation of 5 mm. A microwave generator is attached to one of the wires. As field is ramped, spin waves with progressively different wavevectors are launched across the sample and an output $A = A(H) \exp(ikd)$ is sensed by the receiving wire. A mixer is used to detect the difference in phase between the applied and received microwave fields which changes with the applied field. A 380 Hz field modulation is applied for lock-in detection. In this way, we can measure the change in wavevector as we sweep the field. The geometry of this figure corresponds to the backward volume mode. The resultant spin wave wavevector points in a direction perpendicular to the wire antennas, which in this figure is the same as the magnetic field. The magnets, and therefore the field, can be rotated about an axis perpendicular to the sample plane to allow access to either the backward volume or Damon-Eshbach mode and any angle in between.

*Fig. 3.* Phase detection of spin waves at 6 GHz when the field is parallel to the wavevector, corresponding to the backward volume mode. The ferromagnetic resonance can be observed at around 1405 G. Irregular oscillations can be seen at small field differences from the FMR resonance field due to the spin waves not being launched strictly perpendicular to the wire when the spin wave wavelength is longer than the wire antenna length.28 A full oscillation corresponds to a change in the wavevector of $2\pi/d$, where $d$ is the width of the sample. The vertical dashed red line denotes a break between scaled and unscaled data. The high field side of the vertical dashed line has been scaled by a factor of 50.
fields above the FMR field. As the field is increased from the FMR field, the wavevector initially increases linearly. Using the phase detection measurement technique described in Sec. III, this wavevector increase produces an oscillation in the signal amplitude due to the change of the phase difference between the input microwave signal and the phase of the arriving spin waves at the other end of the sample. An example of this kind of oscillation is seen in Fig. 3. Note that there is a gap that appears between the FMR and the onset of regular oscillations which arises from the effects of diffraction. Initially, we have $kL \ll 1$, where $L$ is the length of the antenna; this results in curved wavefronts leading to phase cancellation across the receiving antenna. But as the field increases further, and the spin wavelength becomes shorter than $L$, we pass over to the regime where $kL \gg 1$ and the wavefront approaches a straight line.

The phase change of a spin wave that propagates across the sample is given by $kd = \phi$, where $d$ is the width of the sample; therefore, a change in $k$ of $2\pi/d$ results in a full oscillation. By tracking these oscillations, we can map out the change in wavevector as a function of field up to a constant offset of wavevector. An example of this behavior for the backward volume geometry is shown in Fig. 4, the total change in wavevector being about $300 \text{ cm}^{-1}$. These data correspond to the low $k$ regime of the overall behavior, where the dispersion relation is essentially linear. Apart from intrinsic attenuation, the amplitude of the signal at larger $k$ is limited by geometrical effects arising from non-parallel edges and accompanying roughness of those edges.
A. Spin wave velocity

While the absolute wavevector is not determined directly through the measurement, the slope of the dispersion relation can be directly measured. If we fit our $H$ vs. $k$ graph, we can determine $\partial H / \partial k$; the spin wave group velocity, $\partial \omega / \partial k$, then follows from the chain rule

$$\frac{\partial \omega}{\partial k}_{|H} = \frac{\partial H}{\partial k} \left( \frac{\partial \omega}{\partial H} \right)_{|k}.$$  

Figure 4 shows a plot of $H$ vs. $k$ which corresponds to a velocity of $-14581$ m/s. This velocity is most closely related to a phase velocity. Due to the fact that the frequency does not go to zero as the wavevector goes to zero it is a modified phase velocity $(\omega - \omega_0)/k$. In the small $k$ regime where the dispersion is linear, this modified phase velocity is equivalent to the group velocity.

According to the theory of Kalinikos and Slavin, the slope of the dispersion relation at small $k$ for the backward volume mode is derived in the Appendix and has the following behavior:

$$v_{\text{pinned}} = 8sM_H \gamma^2 \frac{4\pi M H_0}{\omega} \sin^2 \theta \cos^2 \theta / C_{20} / C_{21},$$  

(3)

where $\theta$ is the angle between the in-plane wavevector and the applied field. Equation (3) assumes fully pinned boundary conditions on both sides of the YIG layer. In the opposite limit of the unpinned boundary condition, these same authors give the following expression for the velocity:

$$v_{\text{unpinned}} = \pi s M H_0 \gamma^2 \frac{4\pi M H_0}{\omega} \sin^2 \theta / C_{20} / C_{21},$$  

(4)

Note that the velocity decreases as the angle between the in-plane wavevector and field increases, ultimately going to zero at a critical angle. The experimentally observed

Figure 7. Angular dependence of the velocity around the DE mode. Using the thickness found in Fig. 5, we plot the expected velocity dependence using Eq. (4) for unpinned spins and Eq. (3) for pinned spins. We find good agreement between the pinned conditions and our data, which is different from the mixed boundary conditions found in the backward volume mode.

![Damon-Eshbach Mode Pinned vs. Unpinned Boundary Conditions](image)

**FIG. 7.** Angular dependence of the velocity around the DE mode. Using the thickness found in Fig. 5, we plot the expected velocity dependence using Eq. (4) for unpinned spins and Eq. (3) for pinned spins. We find good agreement between the pinned conditions and our data, which is different from the mixed boundary conditions found in the backward volume mode.

**FIG. 8.** Energy splitting expressed in the field as a function of higher order BV mode number. The line shows a fit to Eq. (5) using the film thickness found by ellipsometry, $2.84 \mu m$, from which we find an exchange constant of $(5.64 \pm 0.04) \times 10^{-7}$ Oe cm$^2$ and an FMR resonance field of 1405.26 G.

**FIG. 9.** Resonances arising from the exchange split BV modes in a similar field range as Fig. 6. For this measurement, the sample was placed adjacent to a meanderline which was excited at 6 GHz at a level of 25 dBm; the output amplitude was detected with a microwave diode as the field was swept. At the resonant fields for an exchange split BV mode, sharp absorption-derivative features occur in the field modulation data. These data show good agreement with that shown in Fig. 6 but display a run to run uncertainty of a gauss.
behavior of the oscillations for angles within about 20° of the sample axis is well behaved, but for reasons not well understood, the oscillations become more erratic (less periodic) at larger angles, presumably due to the excitation of other geometric modes of our platelet. The resulting data are shown in Fig. 5. Also shown is a comparison of these results against Eqs. (4) and (3) for unpinned and pinned boundary conditions, respectively. From this comparison, we conclude that for this mode, the sample exhibits mixed boundary conditions. This is to be expected due to the different environments at each surface, one with GGG and one exposed to air.

V. DAMON-ESHBACH MODE

When the field is in-plane but perpendicular to \( k \), we encounter oscillations on the low field side of the FMR line associated with the Damon-Eshbach (DE) mode, of which...
Fig. 6 shows an example. Such data can be analyzed using techniques similar to those described in Sec. IV. However, another feature emerges here in connection with the interaction between the DE mode and the higher energy, exchange-split, backward volume modes.

A. Spin wave velocity

Figure 6 shows data obtained using the method of Sec. III when the magnetic field is in-plane but perpendicular to the propagation direction. Similar to the backward volume case, there are oscillations, but in agreement with the predicted dispersion relation, the oscillations occur on the low field side of the FMR line. Using these oscillations, we can determine the spin wave velocity as a function of the angle between the field, \( \mathbf{H} \), and the wavevector, \( \mathbf{k} \). Both unpinned and pinned conditions are plotted in Fig. 7 using Eqs. (4) and (3). In this case, we seem to be most consistent with the predictions for pinned boundary conditions. This is different from the mixed boundary conditions found in the backward volume mode, which may be related to the differing character of the modes (surface vs. bulk). As will be discussed in Sec. V B, there are also nodes due to the avoided crossings between exchange split backward volume modes and the Damon-Eshbach mode where the group velocity goes to zero, which may affect the velocity we find here.

B. Exchange splitting

Apparent in Fig. 6 is the appearance of nodes, which are highlighted by arrows. Additionally, these nodes are apparent in the lower left and upper right of Fig. 11. These nodes correspond to the presence of the higher order backward volume modes that are shifted to larger frequencies through the exchange energy and as a result cross the Damon-Eshbach dispersion relation.\(^{5,13,19-25}\) The field positions of these modes allows us to determine the exchange constant by expressing the energy splitting,\(^7\) \( E_n = \frac{\rho_{ex} \pi^2 n^2}{\mu} \), in terms of the fields \( H_n \) given by

\[
H_n = \frac{f}{g(H_0 + 2nM)} \mu \frac{\rho_{ex} \pi}{s} n^2
\]

equation (5)

for the splitting of the \( n \)th exchange split backward volume mode; here, \( H_0 \) is the FMR resonance field, \( M \) is the saturation magnetization of the sample, \( \rho_{ex}/\mu \) is the exchange parameter, and \( s \) is the film thickness. As noted before, this thickness of the sample was determined through ellipsometry to be 2.84 \( \mu m \), which allows us to fit for the exchange parameter, \( \rho_{ex}/\mu \), and the FMR resonance field using Eq. (5). We find the exchange parameter to be \((8) \ 5.64 \times 10^{-9} \pm 0.04 \times 10^{-9} \ G \) and the FMR resonance field to be 1405.26 \pm 31 G; the latter is consistent with that obtained by direct measurements of the resonance fields, which we discuss next.

In addition to nodes in the phase detection measurement, the exchange split higher order volume modes should contribute sharp resonances in an FMR-like measurement.\(^{23}\) For these measurements, we placed our sample adjacent to a meanderline operating in a transmission mode\(^{24} \) and swept the field. Microwaves are applied to the meanderline, and as the field is swept through a resonance condition, the microwaves are absorbed by the sample. Due to lock-in detection, these resonances appear as a derivative of a Lorentzian. The resonances in question are the FMR resonance and the exchange split mode resonances, which occur below the FMR field as shown in Fig. 9.

Apart from some irreproducibility on the level of a gauss, the meanderline resonance positions agree with those of the nodes observed in the phase detection measurements, which give support to the claim that the nodes pointed out in Fig. 6...

![FIG. 12. Velocity vs. wavevector for a 2.84 \( \mu m \) thick film adjacent to a Cu ground plane for positive field fit with Eq. (8) for the value of the wavevector and the separation between the metal layer and the YIG film. The peak noted agreement between our data and the theory of Bongianni.\(^7\) The spin wave velocity is increased by up to a factor of 7 on the Cu side of the sample compared to the GGG side of the sample. The separation between the metal layer and the YIG film, \( t \), is found through the fit to be 14.8 \pm 0.3 \( \mu m \).](image1)

![FIG. 13. A schematic of the configuration described in Sec. VI B. A 9.72 \( \mu m \) YIG sample grown on GGG has 100 nm of Cu deposited on it and is pressed up against a Cu ground plane. The spacing between the YIG and the Cu ground plane is not well known. The magnetic field is into or out of the plane. The two wire antennas run along opposing sides of the sample. The spin wave propagates from one antenna to the other depending on which has the input microwaves applied to it. This configuration corresponds to the Damon-Eshbach mode and the spin wave propagates on the top or bottom of the YIG layer depending on the direction of the magnetic field and wavevector direction.](image2)
are due to the avoided crossings between the Damon-Eshbach mode and the exchange split backward volume modes.

VI. METALS ON YIG

We can also use our phase detection method to investigate the effects of metals on spin wave propagation. In particular, it may be a useful tool to study the interfacial Dzyaloshinsky-Moriya interaction (DMI) parameter of spin-orbit coupled materials, which should be largest in high Z metals such as Pt. Interfacial DMI should present itself as a difference between forward and backward propagating spin waves, henceforth called nonreciprocity of spin wave propagation. For samples of the thickness reported in Sec. III, iDMI is not the only source of nonreciprocity. The conductivity associated with the metal itself should also induce nonreciprocal spin wave propagation. We will use a simple model of this nonreciprocity. In order to do this, we will go back to Damon and Eshbach theory. In Damon and Eshbach theory, the spin wave amplitude is evaluated through the boundary conditions, both continuity of the spin wave amplitude at the interface between a ferromagnet and the space around it and the spin wave amplitude going to zero at infinite distance from the sample. Instead of the spin wave amplitude going to zero at infinite distance from the sample, we will impose the boundary condition that the spin wave amplitude goes to zero at the surface of the metal layer, which is equivalent to an approximation in which

![Diagram of Double Reversal: GGG/YIG/Cu](image)

**FIG. 14.** Phase detection of spin wave propagation in a GGG (5 mm)/YIG (9.72 μm)/Cu (100 nm) structure pressed up against a Cu ground plane. The positive direction of the wavevector and field is shown in the upper left. Upon reversing either the field or the wavevector, the spin wave propagation characteristics change greatly. Reversing both gives us no change to the propagation characteristics, which we call double reversal. Spin wave reflections are apparent in this sample and can be observed in the upper left and lower right plots as small amplitude high frequency oscillations on top of the main signal that matches the frequency of oscillation in the other two plots.
the metal layer has infinite conductivity. The plane of the sample is the x-y plane and the field is in the x direction. In the Damon-Eshbach geometry, Bongianni\(^2\) derived the following dispersion relation:

\[
e^{-2ki_s} = \frac{1}{2(i\Omega N + \Omega_{el}) + 1} \left[ 1 + (i\Omega N + \Omega_{el})\left( 1 + \tanh(-|k_f|) \right) \right] \left[ 1 - (i\Omega N - \Omega_{el})\left( 1 - \tanh(-|k_f|) \right) \right]. \tag{6}
\]

Here, \(t\) is the separation between the YIG and the metal layer, \(s\) is the thickness of the YIG layer, and \(N\) is \(\pm 1\) depending on which side of the material the spin wave is traveling on (+1 on the metal side and –1 on the side farthest from the metal); the two parameters, \(\Omega\) and \(\Omega_{el}\), are defined as

\[
\Omega = \frac{\omega}{4\pi M}, \quad \Omega_{el} = \frac{H_0}{4\pi M}. \tag{7}
\]

The measurement technique utilized is the same as in Sec. III, but with two different samples. The first was the 2.84\(\mu\)m YIG sample from Secs. IV and V which was pressed up against a Cu ground plane; while the second was a 9.72\(\mu\)m YIG sample on which 100 nm of Cu was deposited. Since the spin wave dispersions in the Damon-Eshbach mode should be different depending on which side the spin wave is traveling on, we show phase detection data for the four combinations of wavevector direction and field direction.

### A. YIG near Cu

We start with a configuration in which the 2.84\(\mu\)m YIG sample is pressed up against a Cu ground plane. A schematic of this configuration is shown in Fig. 10. In this situation, the value of the separation between the YIG and Cu, \(t\), is non-zero but not precisely known. Data were taken in the Damon-Eshbach geometry for both positive and negative wavevectors and field. In order to reverse the wavevector direction, we interchange the launching and receiving antennas. To reverse the field, we reverse the current in the magnet. The data from these four configurations are shown in Fig. 11. An arbitrary direction is chosen to be the positive direction.

The field positions where we expect the FMR are marked with arrows. For either negative wavevector direction or negative field direction, we observe data similar to that of a YIG film with no adjacent metal. This field/wavevector configuration corresponds to the spin wave propagating on the side of the YIG film which is farthest from the Cu ground plane, the GGG side of the sample. Reversing either the wavevector or field direction yields much different data. We no longer observe nodes where the exchange split backward volume modes have avoided crossings with the Damon-Eshbach mode and we note a very large change in the spin wave velocity. Reversing the direction of both the field and wavevector produces data similar to the non-reversed configuration. We plot the velocity against wavevector for positive field and both positive and negative wavevector in Fig. 12. This velocity is not the modified phase velocity of Sec. IV A, but the group velocity is found by taking adjacent peaks in the phase detection data, which corresponds to a change in wavevector of \(\pi/4\), divided by the change in the field, and converting from field to frequency using the Kittel equation. We fit these velocities using Eq. (6) for the absolute value of the wavevector and the separation between the metal layer and the YIG film. We find good agreement between the theory and our results. Discrepancies between the fit and the data on the GGG side of the sample are due to the exchange split backward volume mode avoided crossings where the group velocity goes to zero.

We find a large discrepancy in the spin wave velocity between waves traveling on the side nearest the metal and farthest from the metal, up to a factor of 7. The fitted separation between the metal layer and the YIG film is found to be 14.8 \(\pm\) 0.3\(\mu\)m. This separation is expected to be larger than the likely actual separation due to the fact that the theory we are using assumes infinite conductivity and infinite thickness of the metal layer.

### B. Cu deposited on YIG

We deposited 100 nm of Cu on a 9.72\(\mu\)m thick YIG film via electron beam evaporation and pressed the Cu side of the sample up against a Cu ground plane (Fig. 13). Once again, we plot the four configurations of field and wavevector direction, which is shown in Fig. 14. For this thickness of the YIG film, we note that the spin waves launched from one side of the sample can be reflected from the receiving end and then reflected again from the initial launching side. For this reason, we observe an additional oscillation arising from the
contribution from a signal with a different group velocity. Since this is a derivative measurement, the slower group velocity signal, which shows up as faster oscillations, appears with an amplitude that is increased by the ratio of the frequencies.

Again, we plot the velocity vs. wavevector and fit it with Eq. (6) for the wavevector assuming \( t = 0 \) as shown in Fig. 15. We find that the theory of Bongianni overestimates the velocity of spin waves propagating on the Cu side of the YIG film. Since this is most likely due to the finite thickness and conductivity of Cu, we overlay an additional fit assuming a nonzero \( t \). The skin depth of Cu at 6 GHz is 0.841 nm, but we find an effective separation of 7.48 ± 0.72 μm.

VII. CONCLUSION

Here, we have described a simple method to map out the dispersion relation of spin waves with respect to the field and the field angle. It should also be applicable to other material systems having sufficiently small damping. Basic properties of samples such as film thickness and the exchange parameter can be obtained. This method can also be extended to measure the true dispersion relation using frequency sweeps.

With better control of edge parallelism and roughness, larger wavevectors should be achievable. Recent progress in the preparation and characterization of nanometer thickness films can open the way to study spin wave propagation in the very thin limit.

We have demonstrated nonreciprocal spin wave propagation due to the proximity of a metal layer next to a YIG film. For nonzero separation of the metal layer, we can fit our data to the theory of Bongianni with an effective thickness, but for zero separation, we find poor agreement. Allowing for an effective separation gives a better fit but is still not sufficient. Recently, the effect of finite conductivity and thickness of the metal layers on spin wave propagation has been considered, although the associated formalism is necessarily quite complex. The method used in this paper can be used to test these theoretical predictions.

If finite conductivity and thickness effects can be accounted for, it would be desirable to extend the theory to include spin-orbit coupling effects such as the Dzyaloshinsky-Moriya interaction and related parameters, for which metal nonreciprocity is a background.

Finally, it would be interesting to consider an adjacent superconducting layer, such as Nb, where it may be possible to make a precision determination of the London penetration depth.

ACKNOWLEDGMENTS

This research was supported by the U.S. Department of Energy (DOE) (Grant No. DE-SC0014424).

APPENDIX: DERIVATION OF PINNED AND UNPINNED BOUNDARY CONDITION LOW K VELOCITY

In this appendix, we will derive the pinned and unpinned boundary condition velocity equations (3) and (4). The following equation was given by Kalinikos and Slaving:

\[
\omega_1^2 = (\omega_H + \alpha \omega_M k_n^2) (\omega_H + \omega_M k_n^2 + \omega_F P_{nn}),
\]

where \( \omega_H = \gamma H \), \( \omega_M = \gamma 4\pi M \), \( k_n^2 = k_n^2 + (n\pi/s)^2 \), \( s \) is the thickness of the ferromagnetic layer, \( k_n \) is the in-plane component of the wavevector, and \( \alpha/\gamma = \rho_{ex}/\mu \).

\[
F_{nn} = P_{nn} + \sin^2 \phi \left[ 1 - P_{nn} (1 + \cos^2 \theta) + \omega_M P_{nn} (1 - P_{nn} \sin^2 \theta) \right],
\]

where \( P_{nn} \) is determined by the pinning condition and \( \phi \) is the out-of-plane angle (\( \phi = 90^\circ \) corresponds to in-plane geometry) and \( \theta \) is the in-plane angle (\( \theta = 90^\circ \) corresponds to DE geometry). To see the exact form of \( P_{nn} \), check the appendix for reference. The lowest order terms of \( P_{nn} \) with respect to the in-plane wavevector \( k_n \) for each pinning condition are

\[
P_{low} = \begin{cases} P_H = \frac{4k_n s}{\pi^2} & \text{for totally pinned surface spins,} \\ P_{00} = \frac{k_n s}{\pi^2} & \text{for totally unpinned surface spins,} \end{cases}
\]

where \( n = 0 \) for totally unpinned surface spins and \( n = 1 \) for totally pinned surface spins. Now, we restrict our geometry to an in-plane wavevector, which means \( \phi = 90^\circ \). Then, for the pinned case,

\[
F_{11} \approx 1 - \frac{4k_n s}{\pi^2} \cos^2 \theta + \frac{4\pi M}{\gamma} \frac{4k_n s \sin^2 \theta}{\gamma H} (\gamma H + \gamma / (\pi s)^2).
\]

For the case ferromagnet thicknesses of a few microns and microwave frequencies of a few GHz, which includes the parameters we used, \( \gamma H \gg \gamma (\rho_{ex}/\mu)(\pi/s)^2 \). Therefore,

\[
F_{11} \approx 1 + \frac{4k_n s}{\pi^2} \left( \frac{4\pi M}{H} \sin^2 \theta - \cos^2 \theta \right)
\]

for totally pinned spins and

\[
F_{00} \approx 1 + \frac{k_n s}{2} \left( \frac{4\pi M}{H} \sin^2 \theta - \cos^2 \theta \right)
\]

for unpinned spins. We now linearize Eq. (A1) using each pinning condition (A5) and (A6). Starting with pinned boundary conditions

\[
\omega_1 \approx \gamma \sqrt{H(4\pi M)} \left[ 1 + k_n \frac{8sM}{\pi H} \left( \frac{4\pi M}{H} \sin^2 \theta - \cos^2 \theta \right) \right]
\]
and similarly for unpinned boundary conditions

$$\omega_0 \approx \gamma \sqrt{H(H + 4\pi M)} \left[ 1 + k_{\text{in}} \frac{\pi s M}{H + 4\pi M} \left( \frac{4\pi M}{H} \sin^2 \theta - \cos^2 \theta \right) \right],$$

(A8)

whose coefficients of the linear $k$ term are equivalent to Eqs. (3) and (4), respectively.

REFERENCES