High-Order Moments and Extreme Value Approach for Value-at-Risk

Abstract

We modify a two-step approach by McNeil and Frey (2000) for forecasting Value-at-Risk (VaR). Our approach combines the asymmetric GARCH (GJR) model that allows the high-order moments (i.e., skewness and kurtosis) of the skewed generalized t (SGT) distribution to rely on the past information set to estimate volatility, and the modified Hill estimator (Huisman et al., 2001) for estimating the innovation distribution tail of the GJR model. Using back-testing of the daily return series of 10 stock markets, the empirical results show that our proposed approach could give better one-day VaR forecasts than McNeil and Frey (2000) and the GJR/GARCH models with alternative distributions. In addition, our proposed approach also provides the accuracy of expected shortfall estimates. The evidence demonstrates that our proposed two-step approach that incorporates the modified Hill estimator into the GJR model based on the SGT density with autoregressive conditional skewness and kurtosis provides consistently accurate VaR forecasts in the short and longer sample periods.

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