High-Order Spectral Stochastic Finite Element Analysis of Stochastic Elliptical Partial Differential Equations

Guang Yih Sheu¹,²
¹Department of Accounting and Information System, Chang-Jung Christian University, Gueiren, Tainan, Taiwan
²Department of Civil Engineering, Feng-Chia University, Taichung, Taiwan
Email: xsheu@hotmail.com

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ABSTRACT

This study presents an experiment of improving the performance of spectral stochastic finite element method using high-order elements. This experiment is implemented through a two-dimensional spectral stochastic finite element formulation of an elliptic partial differential equation having stochastic coefficients. Deriving this spectral stochastic finite element formulation couples a two-dimensional deterministic finite element formulation of an elliptic partial differential equation with generalized polynomial chaos expansions of stochastic coefficients. Further inspection of the performance of resulting spectral stochastic finite element formulation with adopting linear and quadratic (9-node or 8-node) quadrilateral elements finds that more accurate standard deviations of unknowns are surprisingly predicted using quadratic quadrilateral elements, especially under high autocorrelation function values of stochastic coefficients. In addition, creating spectral stochastic finite element results using quadratic quadrilateral elements is not unacceptably time-consuming. Therefore, this study concludes that adopting high-order elements can be a lower-cost method to improve the performance of spectral stochastic finite element method.

Keywords: Spectral Stochastic Finite Element Method; Generalized Polynomial Chaos Expansion; High-Order Elements

1. Introduction

A stochastic partial differential equation is a partial differential equation having stochastic coefficients or forcing terms. Problems expressed as stochastic partial differential equations include such as population dynamics and elastostatics with the uncertainty in the spatial variability of mechanical properties. The spectral stochastic finite element method [1] may be the most popular numerical tools for solving stochastic partial differential equation. Briefly, deriving a spectral stochastic finite element formulation couples a deterministic finite element formulation and representations of those stochastic coefficients and forcing terms. These representations of stochastic forcing terms and coefficients are derived by such as generalized polynomial chaos [2] and Karhunen-Loève expansions [3].

Numerous spectral stochastic finite element formulations are available for some branches of science and engineering. References [4,5] are two recent examples. Nevertheless, the performance of spectral stochastic finite element method is not always satisfactory. For example, the spectral stochastic finite element method predicts less accurate mean values or standard deviations of random fields than the spectral stochastic meshless local Petrov-Galerkin method does [6]. Similar experiences bring about the motive for improving the performance of spectral stochastic finite element method.

Since a spectral stochastic finite element formulation contains a deterministic finite element solution and random field representations, improving the performance of spectral stochastic finite element method may be through adopting more accurate deterministic finite element formulation or random field representations. Available studies (e.g. [1]), which were devoted to evaluate the performance of spectral stochastic finite element method, seem to focus on the latter method. Experiences of applying a spectral stochastic finite element formulation using high-order elements seem to be unavailable. After a web search, only Ngah and Young (2007) [7] had adopted quadratic quadrilateral elements to predict the...