Determining the adaptive decision zone of discrete lot sizing model with changes of total cost

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\textbf{A B S T R A C T}

The Economic Order Quantity (EOQ) zone is beneficial for giving some latitude in picking the lot sizes in a continuous time inventory problem, but it is not suitable for a discrete time inventory problem, the discrete lot sizing (DLS) problem. In this paper, a novel enumeration method is proposed and coded as a user-friendly computerized scheduling system to “visualize” the complex DLS problems by projecting the entire feasible solutions on a two dimension space, where setup frequency and total cost are placed on the horizontal and the vertical axis respectively. First, the zone around the optimal solution in the DLS problem is demonstrated always smooth and this phenomenon is defined as DLS zone in which giving a small penalty cost from the optimal solution brings several alternative solutions for picking the lot sizes. Second, even if the penalty costs are changed, the computerized scheduling system is able to determine the adaptive decision zone and find the included alternative solutions. The flexibility in picking the lot sizes for discrete time inventory problems is significantly enhanced since decision makers are enabled to choose a preferable solution from the DLS zone.

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1. Introduction

From the perspective of the continuous time scale, constant demand rate, and infinite planning horizon, Harris (1913) introduced the Economic Order Quantity (EOQ) model. The total cost curve of EOQ model is available to reflect the total costs corresponding to various order quantities and it is usually a shape of bowl (Solomon, 1959). Most importantly, a region around the lowest point \((Q)\) on the total cost curve is relative flat and this phenomenon is the well-known “EOQ zone”, displayed in Fig. 1 (Stevenson, 2002). From practical standpoint, the EOQ zone gives some latitude in picking the lot sizes because the change in total cost among \(Q\) and a number of order quantities included in the “zone” is not too much. Therefore, the EOQ zone is an adaptive decision zone in which there are several alternative solutions for decision making. From academic standpoint, Solomon (1959) was motivated by the EOQ zone to introduce the mathematical approach for defining the \textit{economic lot size range}. Decision makers could set an acceptable penalty cost in advance and they would produce anywhere within a range of quantities relatively. However, the EOQ zone is only beneficial for giving some latitude in picking the lot sizes in a continuous time inventory problem but not suitable for a discrete time inventory problem, the material requirement planning (MRP) problem.

MRP is an approach used in production scheduling to determine the required parts and materials for end items (Fakhrzad & Khandemi Zare, 2009). MRP system was introduced in the 1950s in US and it had received widespread acceptance in enterprises (Sum, Png, & Yang, 1993). Newman and Sridharan (1992) undertook a comprehensive survey of US companies including machine tools, defense electronics, medical equipment, automobile, plastics, computers, components, and furniture. Their survey results indicated that MRP was the most widely used system for production planning and control (56% of the companies reported using a MRP system).

From the perspective of discrete time scale, dynamic demand, and finite planning horizon, Wagner and Whitin (1958) firstly introduced a standard forward dynamic programming formulation to conduct the discrete lot sizing (DLS) problem in MRP system. Wagner and Whitin's model could be adopted to get the optimal production plan (PP) in a single-stage environment. Then, in a multi-stage environment, Zangwill (1969) proposed a backward recursive algorithm to find the optimal PP set (PPS) for the DLS problem with a serial production structure (each stage has at most one direct predecessor and one immediate successor). By virtue of the optimal PP or PPS (in terms of a single- or multi-stage version), decision makers are enabled to determine how many quantities have to be produced in which periods at the operation stage.

Since solving the DLS problems is particularly formidable (Bahl, Ritzman, & Gupta, 1987), developing the quantitative approaches...
for finding an optimal or suboptimal solution efficiently was always a main stream in the past literature. As only one solution can be provided to decision makers, it is the "stationary strategy" for decision making. In practice, however, decision makers desire to know more than an optimal or suboptimal solution, but the flexibility in picking the lot sizes for the DLS problems was given relatively little attention.

The major purposes of this work are to: (1) explore whether an adaptive decision zone as well as a phenomenon of EOQ zone exists in the DLS problems; (2) find the included alternative solutions for decision making in the DLS problems and to bring about some search results are available to fill the gap regarding the flexibility when the total cost of the optimal solution is changed. The re-

2. Notations and problem statement

2.1. List of notations

The following notations are adopted in this paper.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>(m)</td>
<td>stage index ((m = 1, 2, \ldots, M))</td>
</tr>
<tr>
<td>(t)</td>
<td>period index ((t = 1, 2, \ldots, T))</td>
</tr>
<tr>
<td>(s_{m,t})</td>
<td>setup cost in period (t) at stage (m)</td>
</tr>
<tr>
<td>(h_{m,t})</td>
<td>unit inventory holding cost from period (t) to period (t + 1) at stage (m)</td>
</tr>
<tr>
<td>(d_{m,t})</td>
<td>demands in period (t) at stage (m)</td>
</tr>
<tr>
<td>(q_{m,t})</td>
<td>the produced quantities in period (t) at stage (m)</td>
</tr>
<tr>
<td>(l_{m,t})</td>
<td>the inventory level at the end of period (t) at stage (m)</td>
</tr>
<tr>
<td>(c_{m,m+1})</td>
<td>(c) unit of demands have to be yielded at stage (m + 1) in order to produce one unit of demands at stage (m) namely &quot;production ratio&quot;</td>
</tr>
<tr>
<td>(\delta_{m,t})</td>
<td>Boolean variable: (\delta_{m,t} = 1) indicating a production policy adopted in period (t) at stage (m); otherwise, (\delta_{m,t} = 0) meaning an inventory policy</td>
</tr>
<tr>
<td>(x)</td>
<td>row index corresponding to the Solving-Process data sheet ((x = 1, \ldots, 2^{T-1}))</td>
</tr>
<tr>
<td>(p_{m,x})</td>
<td>the (x)th feasible PP at stage (m)</td>
</tr>
<tr>
<td>(c(p_{m,x}))</td>
<td>total cost of (p_{m,x})</td>
</tr>
<tr>
<td>(\text{ops}_{m,x})</td>
<td>an optimal PPS represented by a set of (x), meaning ([p_{1,x}, p_{2,x}, \ldots, p_{m,x}])</td>
</tr>
<tr>
<td>(R_{m,t}(v,j))</td>
<td>i. at the first round to implement the solution method: (a) set of (x) represents various (p_{1,x}) which can connect with (p_{2,x}); ii. after the first round to implement the solution method: a set of (x) represents various (\text{ops}<em>{m-1,x}) which can connect with (p</em>{m,x}), where (m = 3, 4, \ldots, M) c. (R_{m,t}(v,j)) i. at the first round to implement the solution method: minimal value of various (p_{1,x}) which can connect with (p_{2,x}); ii. after the first round to implement the solution method: minimal value of various (\text{ops}<em>{m-1,x}) which can connect with (p</em>{m,x}), where (m = 3, 4, \ldots, M)</td>
</tr>
<tr>
<td>(\text{mtc})</td>
<td>minimal total cost of the DLS problem</td>
</tr>
</tbody>
</table>

Table 1

Values of the parameters given in the illustrated example.

<table>
<thead>
<tr>
<th>Operation stages</th>
<th>Planning periods</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>January</td>
</tr>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(s_{1,1})</td>
</tr>
<tr>
<td></td>
<td>(h_{1,1})</td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(s_{2,2})</td>
</tr>
<tr>
<td></td>
<td>(h_{2,2})</td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(s_{3,3})</td>
</tr>
<tr>
<td></td>
<td>(h_{3,3})</td>
</tr>
<tr>
<td>The demand of end item (d_{4,5})</td>
<td>10</td>
</tr>
</tbody>
</table>

Fig. 1. An adaptive decision zone (EOQ zone) in a continuous time inventory problem.

Fig. 2. The serial DLS problem represented by a network.
2.2. DLS problems

The DLS problems may be roughly classified into the single- and multi-stage versions. Besides, in order to make the DLS problems fit the real-life circumstances more closely, some practice cases, i.e. capacity constraints (Florian & Klein, 1971), quality discounts (Chyr, Huang, & Lai, 1999), and etc., are taken into account. More details about the DLS problems, readers can refer to the distinguished review papers: Bahl et al., 1987; Brahimi, Dauzere, Najid, & Nordli, 2006; Karimia, Fatemi Ghomia, & Wilson, 2003; Kuik, Salomon, & Van Wassenhove, 1994; Simpson & Erenguc, 1996; Wolsey, 1995.

This study considers the multi-stage DLS problem with a serial production structure without capacity constraints and lead times are zero, but bill-of-material (BOM) concept is taken into account. The inputs at stage \( m + 1 \) are always supplied from the outputs at stage \( m \) immediately (Love, 1972; Zangwill, 1969). Without loss of generality, there are no time lags involved in the transmission of goods from one stage to next and the inventory at the beginning and the inventory at the end of the planning horizon are zero. The discussed multi-stage DLS problem is formulated at Appendix A.1. However, the uncapacitated multi-stage DLS problem can be reducible to \( M \) uncapacitated single-stage problems (Brahimi et al., 2006). One of the well-know solution methods for handling the uncapacitated single-stage DLS problem is Wagner and Whitin’s model (1958), which is presented at Appendix A.2.

Since by assumption \( I_{m,0} = I_{m,T} = 0 \), there are \( 2^{T-1} \) feasible PPs at each stage. Every PP comprises a series of production policies and inventory policies. The serial multi-stage DLS problem can be represented as a combinatorial optimization problem, displayed in Fig. 2. Each node in Fig. 2 denotes a feasible PP and various feasible PPSs are constructed by the feasible PPs or PPSs at stage \( m + 1 \) connecting with the PPS at stage \( m \). For instance, a feasible PP, \( p_{2,2} \), connects with \( p_{2,1} \) to construct the feasible PPS, \( \{p_{2,1}, p_{2,2}\} \), between stage 1 and 2. \( \{p_{2,1}, p_{2,2}\} \) connects with \( p_{2,3} \) to form another feasible PPS, \( \{p_{2,1}, p_{2,2}, p_{2,3}\} \), among stages 1–3. However, to find a globe optimal solution of the network is an optimal solution to multi-stage DLS problem, but some feasible PPs at stage \( m \) are not permitted connecting with the particular feasible PPs at stage \( m + 1 \). Let the label \((d_{m,1}, d_{m,2}, \ldots, d_{m,T})\) represent the feasible PPs. By vir-
due to the zero-inventory property of Wagner and Whitin’s (1958) model, the following Theorem is useful to reduce the amount of the feasible PPSs.

**Theorem 1.** Suppose that a feasible PPS is composed of \((d_{m,1}, d_{m+1,1}, \ldots, d_{m,2}, d_{m+1,2}, \ldots, d_{m,T}, d_{m+1,T})\) and \((d_{m,1}, d_{m+1,1}, d_{m,2}, d_{m+1,2}, \ldots, d_{m,T}, d_{m+1,T})\). When \(d_{m+1,t}\) is satisfied by adopting an inventory policy, \(d_{m,t}\) must also consume the inventory.

**Proof.** Assume that \(d_{m+1,t}\) is complemented by inventory, where \(t = p, p+1, \ldots, q, p > 1, \text{ and } q \leq T\). The total produced quantities of \(d_{m+1,t}\) are \(d_{m+1,t} + d_{m,1} + \cdots + d_{m+1,q} = \sum_{i=1}^{q} d_{m+1,t}\). Therefore, the outputs at stage \(m\) \((d_{m+1,t})\) must be equal to or more than the demands at stage \(m+1\) \((d_{m+1,t+1})\), or else \(d_{m+1,t}\) is impossible to be supplied smoothly. In other words, when \(d_{m+1,t}\) depends on inventory, \(d_{m,t}\) must be also satisfied by adopting an inventory policy. (As the BOM concept is taken into account, \(d_{m,t}\) must be equal to or more than \(c \times d_{m+1,t}\).)

Unfortunately, as the planning period enlarges, the number of the feasible PPSs at each stage is increased exponentially. This situation results in searching the feasible PPSs from stages 1 to \(M\) to become very complicated. However, an algorithm is proposed to find the feasible PPs or PPSs for each particular successive PP efficiently, see Appendix B.

2.4. Complexity in exploring the adaptive decision zone of DLS

The feasible solutions yielded from the EOQ model are a number of order quantities and calculating the total cost of various order quantities is not complicated. If each total cost of the order quantities is projected on a 2D space, where “order quantity” and “total cost” are placed on the horizontal and the vertical axis respectively, connecting the projected points (total costs of the order quantities) completely is the total cost curve of EOQ model.

Conversely, as a serial multi-stage DLS problem is discussed: (1) at the first stage, namely a single-stage DLS problem, there are a large number of PPs; (2) from the second to the last stages, there are a large number of PPSs (see Fig. 2). Therefore, (1) at the first stage, the total costs of the entire feasible PPs are projected on a 2D space, where “setup frequency” (the number of setup included in a feasible PP) and “total cost” are placed on the horizontal and the vertical axis respectively; (2) from the second to the last stages, “optimal cumulative total cost” is placed on the horizontal and the vertical axis rather

![Fig. 5. The profile of optimal total cost curve at the 1st stage.](image-url)
than "total cost" since each feasible PP at these stages is connected by various immediate previous feasible PPs or PPSs. The sum of the minimal total cost of the immediate previous feasible PPs (or PPSs) and the total cost of a particular feasible PP is an optimal cumulative total cost. Connecting the lowest points with various setup frequencies (hereafter optimal points for short) is an optimal total cost curve or an optimal cumulative total cost curve of DLS model in terms of a single-stage or multi-stage problem. The depicted optimal (cumulative) total cost curve can help decision makers to know the variation around the optimal solution (the lowest point on the curve) when the total cost of the optimal solution is changed.

One may notice that the lowest point on the optimal (cumulative) total cost curve is an optimal solution to the DLS problems. Therefore, to ensure finding an optimal solution is a precondition for exploring "whether an adaptive decision zone exists in the DLS problems". Obviously, the proposed heuristic methods failed to satisfy such a precondition. In addition, the conventional optimal methods always specialized in computing an optimal solution efficiently rather than searching the entire optimal points. For example, Wagner and Whitin’s model (Appendix A.2) presented a good recursive relationship between \( g(t'/C_0) \) and the next period \( s_t \) that is useful to attain the optimal solution efficiently but not finding the entire optimal points. In order to attain the optimal points for visualizing the optimal (cumulative) total cost curve of DLS model, an enumeration method is proposed in this paper. The details about the proposed enumeration method are described in Appendix B. In order to make readers understand the implementation of solution procedures for getting the optimal points and the optimal solution, an example is presented in Section 3.

### 3. An example

An example, a serial DLS problem with three stages and 12 periods, is taken to illustrate the proposed enumeration method. The values of parameters given in this example are listed in Table 1. The details about implementing the solution procedures on the Solving-Process data sheet are illustrated with Fig. 3.

The total costs of the entire feasible solutions at each stage are projected on a 3D space (Fig. 4a) and overlapped on a 2D space (Fig. 4b). If the optimal points at each stage are connected, the optimal total cost curve at stage 1 and the optimal cumulative total cost curves at stages 2 and 3 are visualized.
A region around the optimal solution is relatively flat (see Fig. 4) and comprises several alternative solutions with a small penalty cost from the optimal solution. This region is defined as “DLS zone” as well as the phenomenon of EOQ zone. In addition, the validity of the proposed method is verified by employing one of the proposed optimal methods, cost-path algorithm (Chyr, Lin, & Ho, 1990), to solve the same example. The same optimal solution can be also obtained by cost-path algorithm, but cost-path algorithm focuses on computing an optimal solution efficiently rather than attaining the optimal points completely.

4. Adaptive decision zone of DLS model

In production and operations management, “changes of setup cost” are a popular case in research. Porteus (1986) discussed the influence of setup cost on EOQ model; Zangwill (1987) addressed a parametric algorithm for cutting stationary setup cost and from EOQ towards zero-inventory (ZI); Chyr (1996) investigated the effects of varying setup cost in the DLS problems; Chung and Chyr (1997) presented a mathematical approach to demonstrate that Wagner and Whitin’s model was close to ZI as $s_{M} < h_{M} \times d_{M}$, where $M = 1, 2, ..., T$. In this study, Chung and Chyr (1997) result is adopted to be a parameter ($\theta$) for examining the impact of changes of setup cost on the optimal (cumulative) total cost curve in the DLS problems.

Since $s_{M}, h_{M},$ and $d_{M}$ are always positive, $s_{M} < h_{M} \times d_{M}$ is reformulated as follows:

$$0 < \frac{s_{M}}{d_{M} \times h_{M}} \leq 1, \quad M = 1, \ t = 1, \ldots, T.$$  \hspace{1cm} (1)

Set $\theta$ be:

$$\theta = \frac{s_{M}}{d_{M} \times h_{M}}, \quad m = 1, \ldots, M, \ t = 1, \ldots, T.$$  \hspace{1cm} (2)

In order to get more insights into the impact of changes of setup cost on the optimal (cumulative) total cost curve in the DLS problems, various $\theta$ values are adopted to classify the test problems.

$$i < \theta \leq i + 1, \quad i = 1, \ldots, 4.$$  \hspace{1cm} (3)

One may notice that $0 < \theta \leq 1$ is not discussed in this study because it is a special case in the DLS problem.

**Theorem 2.** In the serial multi-stage DLS problem, as $0 < \theta \leq 1$, the optimal solution must be made up of a series of ZI policies.
Proof. Chung and Chyr (1997) demonstrated that as $0 < \theta \leq 1$, a feasible PP with $T$ times of setup (namely node $p_{m-1,2}$ of the network in Fig. 2) must be a minimal total cost among the entire feasible PPs at stage $m$. In addition, $p_{m-1,2}$ can connect with $p_{m+1,2}$ to construct the PPSs that has been demonstrated by Theorem 1. It can be concluded that as $0 < \theta \leq 1$, the optimal solution to the serial multi-stage DLS problem must be made up of a set of PPs with $T$ times of setup, namely a series of ZI policies.

The optimal (cumulative) total cost curves in the serial multi-stage DLS problem are depicted in a set of test problems of size 1, 2, 10, and 30 stages; the length of the planning horizon for the test problems is fixed to 12; and each test problem run 50. Demands, setup costs, and unit inventory holding cost are generated randomly, but they are subject to the range of $\theta$ in each test problem. For simplification, the optimal points ($\text{otc}_c$) in every test problem are standardized as “total cost percent” (TC%) namely $\text{otc}_c / \text{Min} \{\text{otc}_c\}$.

From Fig. 5–8, the box plot is adopted to present the distribution of TC% with a particular setup frequency. To connect the medians of the entire setup frequencies completely is the profile of optimal (cumulative) total cost curves of DLS model. From Fig. 5–8, the following propositions are inferred:

Proposition 1. Whatever the $\theta$ is, in the serial multi-stage DLS problems, the optimal total cost curve is a shape of “bathtub”, but the optimal cumulative total cost curve is a “L-type” scree plot.

Proposition 2. A region around the optimal solution is relative flat and this region is defined as “DLS zone” in which there are several alternative solutions with a small penalty cost from the optimal solution.

Proposition 3. As the stage increases, the DLS zone will be steadier and flatter relatively.

Proposition 4. As setup cost gets higher, the range of the DLS zone will be narrowed down relatively and the optimal solution will move toward less setup frequencies.

Fig. 8. The profile of optimal cumulative total cost curve at the 30th stage.
The propositions of (1) and (2) encourage scholars to develop an efficient method for defining the DLS zone and to find the alternative solutions included in the “zone” (readers may refer to Solomon (1959) work); (3) indicates that ZI policy tends to be an alternative solution in the later operation stages in the serial multi-stage DLS problems; and (4) means that cutting setup cost gets more alternative solutions for decision making. Since giving a small penalty cost from the optimal solution and a number of alternative solutions is not too much. An adaptive decision zone as well as a phenomenon of the EOQ (1959) work); (3) indicates that ZI policy tends to be an alternative solution in the later operation stages in the serial multi-stage DLS problems; and (4) means that cutting setup cost gets more alternative solutions for decision making. The profile of the optimal (cumulative) total cost curve in the DLS problems is shown at Fig. 9.

5. Conclusions

An adaptive decision zone as well as a phenomenon of the EOQ zone is demonstrated always existing in the DLS problems with single- and multi-stage versions and it is defined as DLS zone. Within the DLS zone, the change in total cost among the optimal solution and a number of alternative solutions is not too much. In other words, giving a small penalty cost from the optimal solution brings several alternative solutions for decision making. Since the developed computerized scheduling system is able to determine the adaptive decision zone and find the included alternative solutions completely, decision makers are enabled to select a preferable solution from the DLS zone even if various penalty costs are taken into account. Therefore, the flexibility in picking the lot sizes in a discrete time inventory problem is significantly enhanced. The further advanced research will extend the proposed method to handle the multi-stage DLS problem with a more complex production structure and to explore whether an adaptive decision zone also exists in the problem.

Acknowledgement

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Appendix A. DLS models

A.1. Multi-stage DLS model

The DLS problem with a multi-stage version without capacity constraints can be formulated as follows:

\[ \text{Min} \sum_{m=1}^{M} \sum_{t=1}^{T} \left[ s_{mt}(d_{mt}) + h_{mt}(l_{mt}) \right] \]

subject to the constraints

\[ l_{mt} = l_{mt-1} + q_{mt} - d_{mt}, \]
\[ d_{mt} = c_{m, t-1} \cdot q_{mt-1}, \]
\[ \delta_{mt} = \begin{cases} 0, & \text{if } q_{mt} = 0 \\ 1, & \text{if } q_{mt} > 0 \end{cases} \]
\[ I_{m, 0} = I_{m, T} = 0, \]
\[ q_{mt} \geq 0, \quad l_{mt} \geq 0. \]

A.2. Wagner and Whitin’s model

Let \( g(t) \) be the minimal total cost for periods 1 to \( t \) in the entire feasible PPs at the end of period \( t \). Assume that \( d_t, h_t \) and \( s_t \) is the number of demands, unit inventory holding cost, and setup cost in period \( t \) respectively. Wagner and Whitin (1958) developed a standard forward dynamic programming formulation, see Eq. (10).

\[ g(t) = \text{Min} \left\{ S_t + g(t - 1) \right\} \]
\[ \text{Min} \left\{ g(t - 1) + S_t + \sum_{m=1}^{M} \sum_{k=1}^{l_0} (h_{mk} d_k) \right\}, \]

where \( g(0) = 0, \ g(1) = s_1, \ t = 1, 2, \ldots, T \) and this formulation is only suitable for the DLS problem with a single-stage version.

Appendix B. Enumeration method

In order to “visualize” the complicated DLS problems, an enumeration method is proposed to obtain the entire feasible solutions and they are projected on a 2D space. The method is designed to implement on a database entirely since optimization for DLS problem always encounter heavy computational burden as it includes a large number of stages (Simpson and Erenguc, 1996). The demand for computer memory is reduced significantly by means of the utilization of database storage. Besides, during the computational processes, the multiple stages are partitioned into a series of adjacent stages for handling repeatedly, i.e. \( \text{stage (1 & stage 2)}, \text{stage (1, 2) & stage 3}, \ldots, \text{stage (1, 2, \ldots, M - 1) & stage M} \). Visual Basic 6.0 is adopted to code the proposed method for implementing on Access Database as a user-friendly computerized scheduling system. The details about the solution procedures of the enumeration method are as follows:

(1) Conducting \( \text{stage (1 & stage 2)} \) (at the first round to implement the solution procedures):

**Step 1:** Establish a database for storing the computational and input data

1.1 Set four data sheets, Solving-Process, Setup-Cost, Holding-Cost, and Demand data sheets. The database structure is expressed with Fig. 10.

1.2 Set the length of planning horizons (\( T \)) and the number of stages (\( M \)).

1.3 Input the \( s_{mt}, h_{mt}, \) and \( d_{mt} \) into the Setup-Cost, Holding-Cost, and Demand data sheets.

One may notice that the computational results of the following steps are stored into the Solving-Process data sheet.

**Step 2:** Generate the entire feasible PPs

2.1 Let \( \beta \) a number in decimal system, vary from \( 2^{T-1} \) to \( 2^T - 1 \), where \( T \) was set in Step 1.2. Each \( \beta \) can be converted into a binary number by the coded computer program (Fig. 11) to represent the entire feasible PPs at each stage. Note that the feasible PPs are encoded as a zero-one version in which 1 denotes a production policy and 0 means an inventory policy. Therefore, \( p_{mn} \) is identical to \( p_{m+1,n} \).
2.2 The generated feasible PPs are stored into the Field 2.

Step 3: Find out the feasible PPs or PPSs for each particular successive PP

3.1 A recursive model is constructed as Eq. (11) in which $y$ is

$$4\left(\frac{T}{2}\right)/C_0 w$$

as the planning period is an even number, namely $T = 4, 6, 8$, etc. Otherwise, $y$ is

$$4\left(\frac{T}{2}C_0 \right)/C_0 w$$

namely $T = 3, 5, 7$, etc. The recursive model is formulated as follows.

$$R_{m,x}(v, 1) = \{X\}$$

$$R_{m,x+y}(v, 2) = \{X | X \in [R_{m,x}(v, 1) \cup (R_{m,x}(v, 1) + y)]\}$$

$$R_{m,x+2y}(v, 3) = \{X | X \in [R_{m,x}(v, 1) \cup (R_{m,x}(v, 1) + 2y)]\}$$

$$R_{m,x+3y}(v, 4) = \{X | X \in [R_{m,x}(v, 1) \cup (R_{m,x}(v, 1) + (1 \times y))]\}$$

for $i = 1, 2, 3$, where $R_{m,x}(v, j) + y = \{X + y | X \in R_{m,x}(v, j)\}$.

In Eq. (11), $X$ is a set of x.

3.2 Eq. (11) is implemented on the algorithm in Fig. 12 for finding the feasible PPs or PPSs for each particular successive PP. In addition, since $p_{m,x}$ is identical to $p_{m+1,x}$, $R_{m,x}(v, j)$ will be the same with $R_{m,x}(v, j)$.

3.3 The computed previous feasible PPs or PPSs (a set of $x$) are stored into the Field 3.

Step 4: Calculate the total cost of the feasible PPs at stage 1

4.1 Perform the Decoding Procedure (Fig. 13) for calculating the total cost of each feasible PP.

4.2 The total cost of each feasible PP at stage 1 is stored into the Field 4.

Step 5: Calculate the total cost of the feasible PPs at stage 2

5.1 Perform the Decoding Procedure (Fig. 13) for calculating the total cost of each feasible PP.

5.2 The total cost of each feasible PP at stage 2 is stored into the Field 5.

Step 6: Compare the total cost of the PP included in $R_{2,x}(v, j)$

6.1 Compare $c(pp_{m,x})$ included in $R_{2,x}(v, j)$ to find out a minimal value namely $c(R_{2,x}(v, j))$.

6.2 $c(R_{2,x}(v, j))$ are stored into the Field 6.

Step 7: Obtain the optimal PPSs

7.1 Since the solving process is a forward calculation, the optimal PPSs are:

$$ops_{m,x} = \{ ops_{m-1,x} \} \cup \{ x \},$$

where $ops_{m,x} = \{ x \}$.

In Eq. (12), "ops$_{m-1,x}$ indicates an ops$_{m-1,x}$ with a minimal value among $R_{m,x}(v, j)$.

7.2 The ops$_{m,x}$ are stored into the Field 7.

Step 8: Calculate the optimal cumulative total cost of each feasible PP

8.1 The optimal cumulative total cost for the feasible PPs is the sum of the Field 5 and Field 6 namely Eq. (13).
\( c(\text{ops}_{\text{max}}) = c(p_{\text{max}}) + c(R_{\text{max}}(v,j)), \ m = 2, 3, \ldots, M. \) \hfill (13)

8.2 \( c(\text{ops}_{\text{max}}) \) are stored into the Field 8.

**Step 9:** Find out the optimal points

- If the planning period is odd, namely \( T = 3, 5, 7, \) etc.,
  
  **Begin**
  
  \[ R_{\text{max}}(v,j) = \text{Null}, \ \forall \ m, x, v, j. \]
  
  **For** \( w = 1 \) to \( (T-1)/2 \):
  
  \[ y = 4(i-2w) \]
  
  **If** \( w = 1 \) then
  
  Initial recursive modul (\( v = 0 \)):
  
  \[ R_{1}(0,1) = \{1\} \]
  
  **Compute:**
  
  \[ R_{2}(0,2), R_{2}(0,3), R_{2}(0,4) \]
  
  **Else** if \( w = 2 \) then
  
  \[ \text{For} j = 1 \text{ to } 4: \]
  
  \[ i = 0 \]
  
  \[ v = 4i + j \]
  
  \[ R_{2}(v,j) = R_{2}(i,j) \]
  
  **Compute** the Eq. (11)
  
  **Next** \( j \)
  
  **Else** if \( w > 2 \) then
  
  \[ \text{For} i = \left( \sum_{j=1}^{i-1} 4j \right) \text{ to } \left( \sum_{j=1}^{i} 4j \right) \]
  
  For \( j = 1 \text{ to } 4: \)
  
  \[ v = 4i + j \]
  
  \[ R_{2}(v,j) = R_{2}(i,j) \]
  
  **Compute** the Eq. (11)
  
  **Next** \( i \)
  
  **Next** \( w \)
  
  **End**

- If the planning period is even, namely \( T = 4, 6, 8, \) etc.,
  
  **Begin**
  
  \[ R_{\text{max}}(v,j) = \text{Null}, \ \forall \ m, x, v, j. \]
  
  **For** \( w = 1 \) to \( (T)/2 \):
  
  \[ y = 4(i) \]
  
  **If** \( w = 1 \) then
  
  Initial recursive modul (\( v = 0 \)):
  
  \[ R_{2}(0,1) = \{1\} \]
  
  **Compute:**
  
  \[ R_{2}(0,2) \]
  
  **Else** if \( w = 2 \) then
  
  \[ \text{For} j = 1 \text{ to } 2: \]
  
  \[ i = 0 \]
  
  \[ v = j \]
  
  \[ R_{2}(v,j) = R_{2}(i,j) \]
  
  **Compute** the Eq. (11)
  
  **Next** \( j \)
  
  **Else** if \( w > 2 \) then
  
  \[ \text{For} i = \left( \sum_{j=1}^{i-1} 4j \right) \text{ to } \left( \sum_{j=1}^{i} 4j \right) \]
  
  For \( j = 1 \text{ to } 4: \)
  
  \[ v = 2i + j \]
  
  \[ R_{2}(v,j) = R_{2}(i,j) \]
  
  **Compute** the Eq. (11)
  
  **Next** \( i \)
  
  **Next** \( w \)
  
  **End**

Fig. 12. An algorithm for searching the feasible previous PP or PPSs.

**Procedure on calculating the total cost of each feasible PP**

**Begin**

\[ c(p_{\text{opt}}) = 0 \]

Recall \( p_{\text{opt}}, x_{\text{opt}}, R_{\text{opt}}, \) and \( d_{\text{opt}} \) from the database

**For** \( t = 1 \) to \( T \) (Decode \( p_{\text{opt}} \), going through from the first period to the end of period)

If the "value" in period \( t \) is "1" Then

\[ c(p_{\text{opt}}) = c_{\text{set}} + c(R_{\text{opt}}) \]

\[ j = t \]

Else

\[ c(p_{\text{opt}}) = h_{\text{set}} \times \text{set}(t-j) \times d_{\text{set}} + c(p_{\text{opt}}) \]

End If

**Next** \( t \)

**End**

Fig. 13. A decoding procedure on calculating the total cost of each feasible PP.
a.2 Compare \( c(\text{ops}_2, x) \) with various setup frequencies to get \( \text{motcf}_f \) (optimal points), where \( m = 1, 2, \ldots, T \).

b. If after the first round to implement the solution method:

b.1 Compare \( c(\text{ops}_{m, x}) \) with various setup frequencies to get \( \text{motcf}_f \) (optimal points), where \( m = 3, 4, \ldots, M \); \( f = 1, 2, \ldots, T \).

9.2 If \( m = M \), end running the solution procedures. The minimal total cost of the DLS problem is obtained by:

\[
\text{mtc} = \text{Min} \{\text{motcf}_f\}, \quad f = 1, \ldots, T.
\]  

Otherwise, go to the next round to implement the solution procedures:

2 Conducting the \{stage (1, 2, \& stage 3), \{stage (1, 2, 3, \& stage 4), \ldots, \{stage (1, 2, \ldots, M – 1), \& stage M\} (after the first round to implement the solution procedures):

Repeat to do steps 4–9 at every round for carrying out the solution procedures, but steps 4–6 are modified respectively.

Step 4: Make the data of the Field 8 substitute for the data of the Field 4.

a. If at the second round to implement the solution method:

a.1 Make \( c(\text{ops}_{m, x}) \) substitute for \( c(p_{1, x}) \).

a.2 The \( c(\text{ops}_{m, x}) \) are stored into the Field 4.

b. If after the second round to implement the solution method:

b.1 Make \( c(\text{ops}_{m, x}) \) substitute for \( c(\text{ops}_{m-1, x}) \).

b.2 The \( c(\text{ops}_{m, x}) \) are stored into the Field 4.

Step 5: Calculate the total cost of the feasible PPs at stage \( m \).

5.1 Perform the Decoding Procedure (Fig. 13) for calculating the total cost of each feasible PP.

5.2 The total cost of each feasible PP at stage \( m \) is stored into the Field 5.

Step 6: Compare the total cost of the PPs included in \( R_{2, x}(v, j) \).

6.1 Compare \( c(\text{ops}_{m-1, x}) \) included in \( R_{m, x}(v, j) \) to find out the minimal value namely \( c(R_{m, x}(v, j)) \).

6.2 \( c(R_{m, x}(v, j)) \) are stored into the Field 6.

References


